The Data Analysis – Another Approach

Dear Doug,

I believe that Simon and I have found out why the results in my first table (Please see Report 5 from the last week) are so unstable. The problem is not the fitting formula itself but the way I am using it to fit the data. When I fit both parameters (the beam energy E0 and the central momentum Ec of the spectrometer) simultaneously, the fitted parameters dance around like crazy (If I change the value of relative momentum delta from e.g. -0.01759 to -0.018 the fitted beam energy changes for 3MeV.). However, If I fix the parameter Ec and fit only the beam energy my fit becomes stable.

HRSL - Analysis

Because of these stability problems I have decided to use a different approach to fit my data. First I have analyzed the tantalum data. I have taken together those tantalum runs, that were measured (according to Halog) at the same momentum of the spectrometer, and have calculated their mean value. I have got the following results:

Run #	Kinematics	Set momentum	delta
3064	Kin 1	356	0.0171305
3102	Kin 2	356	0.0178165
3320	Kin 2	356	0.0173817
The mean value 1.7443E-2 \pm 2.834E-4			

Run #	Kinematics	Set momentum	delta
3112	Kin 11	353.8	0.024006
3422	Kin 11	353.8	0.023404
The mean value 2.3705E-2 \pm 3.01E-4			

Run #	Kinematics	Set momentum	delta
3151	Kin 3	351.4	0.0308973

Assuming that the recoil effect is negligible in the tantalum runs, we can simply calculate the ratios between the central momenta of the spectrometer for different kinematics, using the formulas:

$$\frac{E_c^{Kin11}}{E_c^{Kin2}}=\frac{(1+\delta^{Kin2})}{(1+\delta^{Kin11})} \qquad \frac{E_c^{Kin3}}{E_c^{Kin2}}=\frac{(1+\delta^{Kin2})}{(1+\delta^{Kin3})}$$

When I have calculated these ratios I have got a good agreement of gained results with the ratios, calculated from the Hall probe readouts:

Ratio	Value	Hall Probe Ratio
Ec^11 / Ec^2	0.99388	0.9938
Ec^3 / Ec^2	0.98695	0.9871

Once I have determined those ratios, there has been left only one central momentum of the spectrometer (for Kin 2 or Kin 1) to be determined. I have fixed this unknown parameter to an arbitrary value and with it fit my data. I have fitted each set of data (there are six) separately. Because I was fitting only the beam energy, the fits were now stable and in the end all gave consistent results. Afterwards I have started varying my parameter Ec^Kin2 and observed how the sum of the chi^2 of my functions changes with it. I have found its minimum at Ec^2 = 351.3 MeV (See figure below).

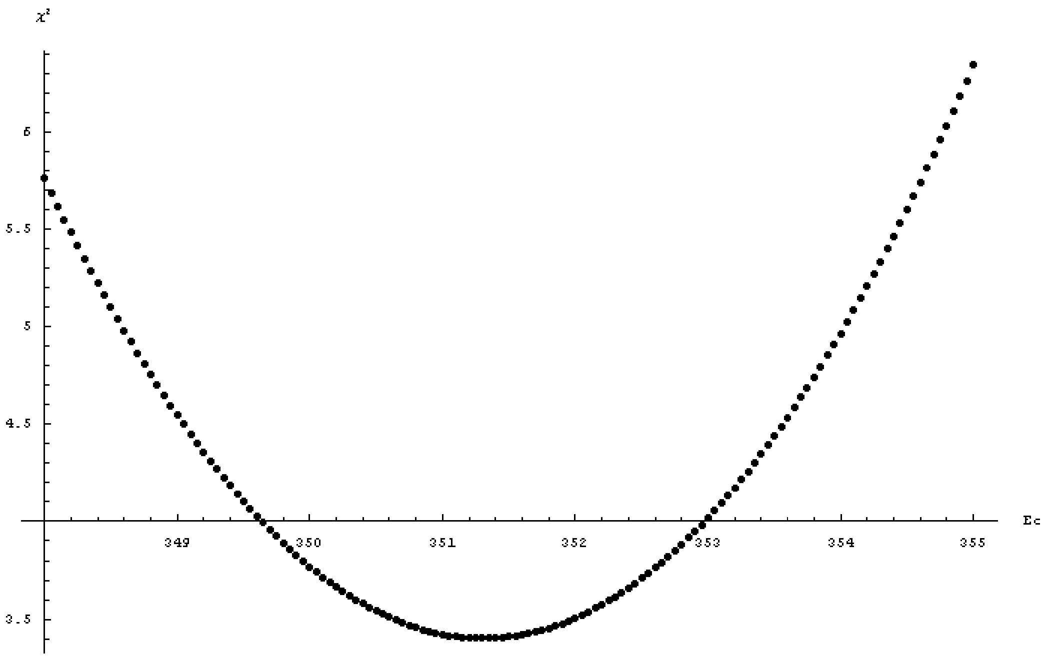

Illustration 1: Graph shows how the chi^2 of the fit changes with the central momentum of the spectrometer.

This has given me the following results for the beam energies and the central momenta of the spectrometer for different sets of data at different kinematic points:

Results from the last week				
Kinematic	**theta**	**chi^2**	**E_beam [MeV]**	**E_c [MeV]**
Kin2, Run2	24 deg	0.089	363.937	357.28
Kin2, Run1	24 deg	0.105	362.83	356.03
Kin11, Run1	28.3 deg	0.0815	361.263	352.321
Kin11, Run2	28.3 deg	0.129	358.529	349.911
Kin3, Run1	32.5 deg	0.0311	357.555	346.371
Kin 1, Rin1	16.0 deg	0.094	344.137	337.958

New Results				
Kinematic	**theta**	**chi^2**	**E_beam [MeV]**	**E_c [MeV]**
Kin2, Run2	24 deg	0.089	357.771	351.30
Kin2, Run1	24 deg	0.105	357.955	351.30
Kin11, Run1	28.3 deg	0.0815	357.963	349.15
Kin11, Run2	28.3 deg	0.129	357.754	349.15
Kin3, Run1	32.5 deg	0.0311	357.934	346.716
Kin 1, Rin1	16.0 deg	0.094	357.802	351.30
Mean value of the beam energy = 357.86 MeV +- 8.904E-2 MeV				

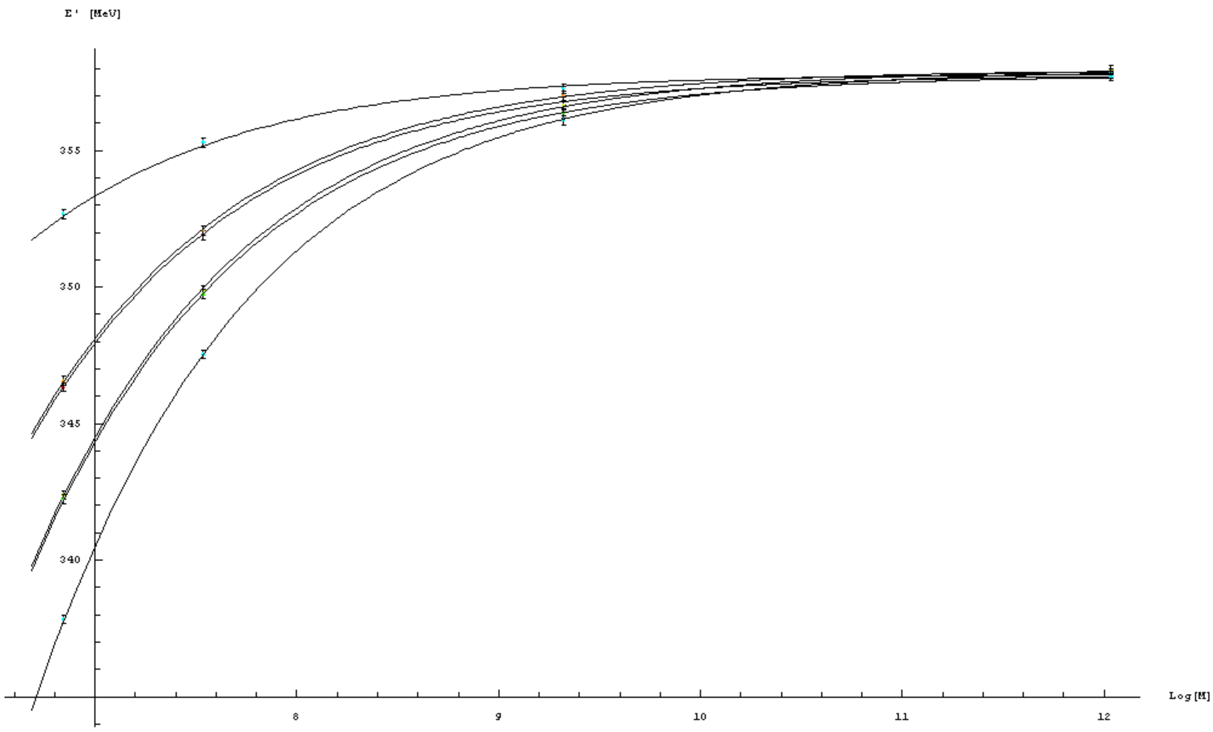

Illustration 2: Graph shows the measured points (transformed into energy units) and their analytical fits.

HRSR – Analysis

I have made the same analysis also for the data, taken with the HRSR spectrometer. In this case, the spectrometer was set to the same momentum for all considered kinematics. As in the HRSL case, I have first analyze the tantalum runs:

Run #	Kinematics	Set momentum	delta
21677	Kin 1	346	0.0269717
21661	Kin 9	346	0.0270123
21699	Kin 10	346	0.02679
21597	Kin 11	346	0.0272082
The mean value 2.69956E-2 \pm 1.48594E-4			

Afterwards I have calculated the ratio between the central momentum of the HRSR spectrometer and the central momentum E_c^Kin2 of the HRSL spectrometer:

$$\frac{E_c^{HRSR}}{E_c^{Kin2}}=\frac{(1+\delta^{Kin2})}{(1+\delta^{HRSR})}$$

I have got the following results:

Ratio	Value	Hall Probe Ratio
Ec^HRSR/ Ec^kin2	0.990698	0.98988

From this table we can see, that the calculated ratio and the ratio, measured by the Hall probe, do not match as well, as they do for the HRSL case. I believe that this difference is caused by the the inconsistency of the magnetic fields inside the magnets of the HRSR spectrometers. As I pointed out in the last week report, the dipole magnet of the HRSR is set to a bigger momentum then quadrupole magnets. This difference gives the wrong ratio of the central momenta of the spectrometers.

From the calculated ratio I was than able to determine the "true" central momentum of the HRSR spectrometer and finally fit the data, measured with this spectrometer. The results of these fits are shown in the table below:

New Results				
Kinematic	**theta**	**chi^2**	**E_beam [MeV]**	**E_c [MeV]**
Kin11, Run2	28.3 deg	8.23614	358.074	348.032
Kin10, Run2	20 deg	2.08904	357.883	348.032
Kin9, Run2	14 deg	1.05254	357.893	348.032
Kin1, Run2	16 deg	1.43816	357.899	348.032
Mean value of the beam energy = 357.937 MeV +- 9.14E-2 MeV				

The Error Estimation

The relation between the beam energy, the central momentum of the spectrometer, the scattering angle of the spectrometer and the measured relative momentum - delta is:

$$\delta(E_{beam}, E_c, \theta, M) = -1 + \frac{\frac{E_{beam}}{E_c}}{1 + \frac{E_{beam}}{M}(1-\cos\theta)}.$$

From this expression I have derived the following formula to estimate the error of my measurements:

$$\sigma_\delta^2 = \left(\frac{\partial \delta}{\partial E_{beam}}\right)^2 \sigma_{beam}^2 + \left(\frac{\partial \delta}{\partial E_c}\right)^2 \sigma_{E_c}^2 + \left(\frac{\partial \delta}{\partial \theta}\right)^2 \sigma_\theta^2 + \sigma_{stat}^2 + \sigma_{Mom.Res.}^2 ,$$

where σ_i are the errors of the parameters that δ depends on. I have estimated these terms in the following way:

1.) The error of the central momentum of the spectrometer has been estimated from the fluctuations of the magnetic field inside the magnets of the HRSL spectrometer (see figure below):

The Magnet	**Mean value [GeV]**	**Standard deviation [GeV]**
Q1	0.3538	1.529 E-4
Q2	0.3538	3.1414E-5
D1	0.3538	1.6431E-5
Q3	0.3538	2.255E-5
Total Error of Q2+Q3+D1 = 4.202E-5 GeV		

I have excluded the Q1 data from the total error estimation, because the formula that I have been using to calculate the momentum of the Q1 magnet from the measured magnetic field is not totally valid in this energy regime (see report from John LeRose) .

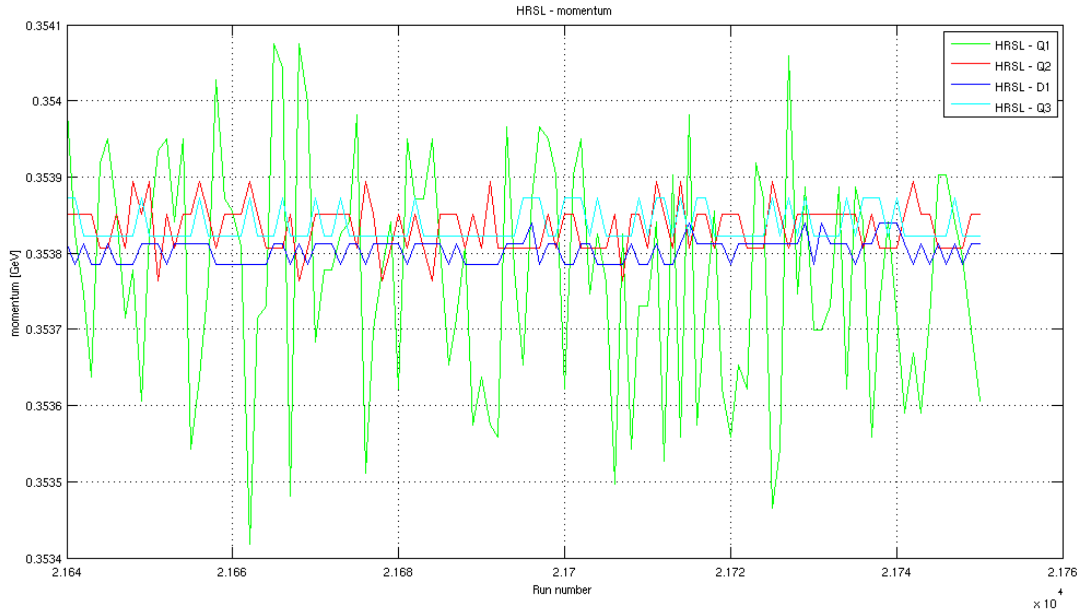

Illustration 3: Graph shows the fluctuation of the momentum/magnetic field inside magnets of the spectrometer HRS

2.) The error of the beam energy was estimated using the Tiefenbach data (see figure below). Assuming that the Tiefenbach correctly describes the relative changes in the beam energy, the fluctuations of the beam energy are approximately : $\sigma_{beam} = 0.1686\, MeV$

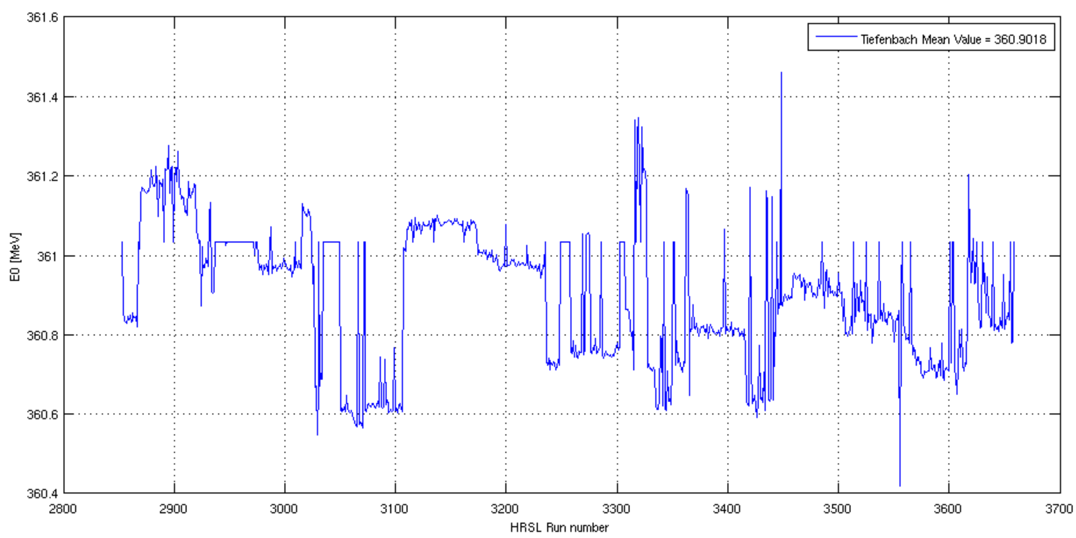

Illustration 4: Fluctuations of the Beam energy, measured with the Tiefenbach.

3.) The uncertainty in the spectrometers angle determination is approximately $\sigma_{\theta} = 0.2\, mrad$ (I have taken these data from the survey report.)

4.) The statistical uncertainties of our measurements are much smaller (of order 10^-7) then systematical uncertainties (of order 10^-5 - 10^-4). Therefore have I decided to neglect these terms.

5.) The last term comes from the finite momentum resolution of the HRS spectrometer and is estimated to be $\sigma_{Mom.\ Res} = 4.2467\text{E-}5$