Ratios between the central momenta of the spectometer

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Up until now I have been using the following formula to calculate the ratios between the central momenta of the spectrometer in order to stabilize my fitting functions:

$$\frac{(1+\delta^1)}{(1+\delta^i)} = \frac{E_c^i}{E_c^1}, \tag{1}$$

where $i = 2, 3, 4, ...$ and characterizes different kinematics. In this fomula I have neglected the recoil of the Ta and assumed that beam energy is constant the whole time. When I looked at the Tiefenbach data, I quickly realized that this is not true. Therefore I needed a new formula to calculate my ratios, which would consider tantalum recoil correction and beam energy changes between different kinematics. Simon and I have used the following procedure to calculate these momentum ratios:

Let us assume that the true beam energy E_0 and the Tiefenbach value E_T are connected through the following formula:

$$E_0 = aE_T + b, \tag{2}$$

where a and b are unknown constants. Now let's assume that the Tiefenbach gives relatively correct results ($a \approx 1$), but is not absolutely calibrated ($b \neq 0$):

$$E_0 = E_T + b, \tag{3}$$

Now we can calculate the ratio beween the energies of scattered electrons of a tantalum target from different kinematic settings:

$$\frac{(1+\delta_1)E_c^1 + \Delta E_{Ta}}{(1+\delta_2)E_c^2 + \Delta E_{Ta}} = \frac{(E_T^1 + b)}{(E_T^2 + b)} \frac{1 + \frac{E_T^2 + b}{M}(1-\cos\theta_2)}{1 + \frac{E_T^1 + b}{M}(1-\cos\theta_1)} \tag{4}$$

Using the Taylor expansion we get:

$$\frac{(1+\delta_1)E_c^1 + \Delta E_{Ta}}{(1+\delta_2)E_c^2 + \Delta E_{Ta}} = \frac{(E_T^1 + b)}{(E_T^2 + b)} \left(1 + \frac{E_T^2 - E_T^1}{M} - \frac{1}{M}(E_T^2\cos\theta_2 - E_T^1\cos\theta 1) - \frac{b}{M}(\cos\theta_2 - \cos\theta_1)\right) \tag{5}$$

The second and the fourth term in the expansion are much smaller then the third term, therefore we can neglect them. This than gives us:

$$\frac{(1+\delta_1)E_c^1 + \Delta E_{Ta}}{(1+\delta_2)E_c^2 + \Delta E_{Ta}} = \frac{(E_T^1 + b)}{(E_T^2 + b)} \left(1 + -\frac{1}{M}(E_T^2\cos\theta_2 - E_T^1\cos\theta 1)\right) = \frac{(E_T^1 + b)}{(E_T^2 + b)}\kappa \tag{6}$$

Now let's take a look at the first term on the right side of the equation. We can expend this term as well:

$$\frac{(E_T^1 + b)}{(E_T^2 + b)} = 1 + \frac{E_T^1 - E_T^2}{E_T^2 + b} = 1 + \frac{E_T^1 - E_T^2}{E_T^2} - \frac{E_T^1 - E_T^2}{E_T^2}\frac{b}{E_T^2} \tag{7}$$

Here we can neglect the last term, because it is two order of magnitude smaller the the second one. This than gives us:

$$\frac{(E_T^1 + b)}{(E_T^2 + b)} = 1 + \frac{E_T^1 - E_T^2}{E_T^2 + b} = 1 + \frac{E_T^1 - E_T^2}{E_T^2} = \Omega \tag{8}$$

When we use this in our main equation we get:

$$\frac{(1+\delta_1)E_c^1 + \Delta E_{Ta}}{(1+\delta_2)E_c^2 + \Delta E_{Ta}} = \Omega\kappa \tag{9}$$

From this equation we can now calculate the ratios between the central momenta of the spectrometer:

$$\frac{E_c^1}{E_c^2} = \Omega\kappa\frac{1+\delta_2}{(1+\delta_1)} + \frac{\Delta E_{Ta}}{E_c^2(1+\delta_1)}(\Omega\kappa - 1) \tag{10}$$

It again turns out that the last term in the right side of the equation is much smaller than the first one, so we can neglect it. This gives us the final equation:

$$\frac{E_c^1}{E_c^2} = \Omega\kappa\frac{1+\delta_2}{(1+\delta_1)} \tag{11}$$

I have used this formula to calculate new ratios between the central momenta of the spectrometer and got the following results:

Ratio $\frac{E_c^i}{E_c^1}$	Scat.Angle	Value
HRSL-2	24.0	1.00047
HRSL-3	28.3	0.995165
HRSL-4	28.3	0.994785
HRSL-5	32.5	0.988508
HRSL-6	16.0	1.00063
HRSR-1	28.3	0.991122
HRSR-2	20.0	0.991621
HRSR-3	14.0	0.991821
HRSR-4	16.0	0.991951

With these ratios I was than able to fit my data again and got the following results. I have compared these results to the Tiefenbach values. Results are arranged from the bigest beam energy to the smallest.

HRS-L			
Kin #	Tiefenbach	Kin #	Fitted energies
5	361.0843	5	363.019
3	361.081	3	362.956
4	360.716	4	362.649
2	360.6596	6	362.483
1	360.617	1	362.425
6	360.592	2	362.366

HRS-R			
Kin #	Tiefenbach	Kin #	Fitted energies
4	360.872	4	362.7
3	360.829	3	362.637
1	360.719	2	362.61
2	360.717	1	362.553