

for momentum transfers below 400 MeV/c, although one cannot generalize this statement to all inelastic cases (see below for examples where larger effects are seen).

An alternative method for studying nuclear structure using unpolarized or polarized electrons involves the inelastic ($e, e'\gamma$) reaction. In such experiments, the target nucleus is *not* polarized, but the photon resulting from the de-excitation of the excited state is detected in coincidence with the scattered electron (if polarized electrons are used, then the polarization of the photon must also be measured for new information to be obtained). The analysis for this process is very similar to the formalism developed in Section 2 for electron scattering from polarized nuclei [62], and ($e, e'\gamma$) studies may complement (e, e') experiments for inelastic transitions for which only small effects are observed. This particular transition for ^{13}C happens to be such a case.

(iii) *Elastic Scattering: $J_i = J_f = 1$*

The possible multipoles are now given by

$$F_0(q) = (1/\sqrt{3}) \langle 1 \| \hat{M}_0(q) \| 1 \rangle, \quad \text{C0} \quad (3.12a)$$

$$F_1(q) = (1/\sqrt{3}) \langle 1 \| i\hat{T}_1^{\text{mag}}(q) \| 1 \rangle, \quad \text{M1} \quad (3.12b)$$

and

$$F_2(q) = (1/\sqrt{3}) \langle 1 \| \hat{M}_2(q) \| 1 \rangle, \quad \text{C2} \quad (3.12c)$$

and we have that Σ_0^{fi} is given by Eq. (3.3) with $F_L^2(q)^{\text{fi}} = F_0^2(q) + F_2^2(q)$ and $F_T^2(q)^{\text{fi}} = F_1^2(q)$. Thus, with unpolarized scattering, only the sum of the squares of the C0 and C2 contributions can be determined, rather than the individual Coulomb form factors themselves. With polarized targets, however, we have that

$$\begin{aligned} \Sigma_{\text{fi}} = \Sigma_0^{\text{fi}} \{ & 1 + P_2(\cos \theta^*) R_2^0(q, \theta_e)_{\text{fi}} + P_2^1(\cos \theta^*) \cos \phi^* R_2^1(q, \theta_e)_{\text{fi}} \\ & + P_2^2(\cos \theta^*) \cos 2\phi^* R_2^2(q, \theta_e)_{\text{fi}} \}, \end{aligned} \quad (3.13a)$$

and

$$\Delta_{\text{fi}} = \Sigma_0^{\text{fi}} (P_1(\cos \theta^*) R_1^0(q, \theta_e)_{\text{fi}} + P_1^1(\cos \theta^*) \cos \phi^* R_1^1(q, \theta_e)_{\text{fi}}), \quad (3.13b)$$

where

$$R_2^0(q, \theta_e)_{\text{fi}} = -f_2^{(i)} \{ v_L (2\sqrt{3} F_2(F_0 + (\sqrt{2}/4) F_2)) + v_T (\frac{1}{2} \sqrt{\frac{3}{2}} F_1^2) \} / F^2(q, \theta_e)_{\text{fi}}, \quad (3.14a)$$

$$R_2^1(q, \theta_e)_{\text{fi}} = v_{\text{TL}} f_2^{(i)} \{ (3/\sqrt{2}) F_1 F_2 \} / F^2(q, \theta_e)_{\text{fi}}, \quad (3.14b)$$

$$R_2^2(q, \theta_e)_{\text{fi}} = v_{\text{TT}} f_2^{(i)} \{ \frac{1}{4} \sqrt{3/2} F_1^2 \} / F^2(q, \theta_e)_{\text{fi}}, \quad (3.14c)$$

$$R_1^0(q, \theta_e)_{\text{fi}} = -v_T f_1^{(i)} \{ (3\sqrt{2}/4) F_1^2 \} / F^2(q, \theta_e)_{\text{fi}}, \quad (3.14d)$$

and

$$R_1^1(q, \theta_e)_R = -v_{TL} f_1^{(i)} \{2\sqrt{3}F_1(F_0 + (\sqrt{2}/4)F_2)\}/F^2(q, \theta_e)^R, \quad (3.14e)$$

using Table C12 in Appendix C. Again, if we assume 100% polarization, then the rank-1 and -2 Fano tensors are $f_1^{(i)} = 1/\sqrt{2}$ and $f_2^{(i)} = 1/\sqrt{6}$, respectively.

First, let us consider the information which may be determined in the absence of polarized electrons. As can be seen, a measurement of R_2^0 , either as here with a polarized target or by measuring the recoil polarization, together with the longitudinal and transverse form factors as determined without having any nuclear polarization, will allow the extraction of the C0 and C2 form factors separately. The usual Rosenbluth separation will give us $F_1(q)$ (up to an arbitrary sign), and the relative sign between the M1 and the C0 and C2 multipoles can be determined by measuring R_2^1 . Thus, it follows that it is in general not necessary to have knowledge of A_R to determine all of the multipoles (see the general discussion in Sect. 2 and in Appendix B). Note that R_2^2 (and also R_1^0) only involves $(F_1)^2$, and so just provides a test of the consistency of the measurement; alternatively, some fitting procedure involving all of the reduced response functions $\mathcal{W}_J^K(q)$ could be used to determine the entire set of form factors.

The fundamental example of a spin-1 nucleus is of course the deuteron (${}^2\text{H}$). The usual definitions of the charge, magnetic, and quadrupole form factors $G_C(q)$, $G_M(q)$, and $G_Q(q)$ prevalent in previous papers are related to our definitions by

$$\sqrt{4\pi}F_0 = (1 + \tau) G_C, \quad (3.15a)$$

$$\sqrt{4\pi}F_1 = -(2/\sqrt{3}) \sqrt{\tau(1 + \tau)} G_M, \quad (3.15b)$$

and

$$\sqrt{4\pi}F_2 = (2\sqrt{2}/3) \tau(1 + \tau) G_Q, \quad (3.15c)$$

where again we note that our form factors only involve the contribution from the leading-order term in the nonrelativistic expansion for the nuclear current (see the discussion above in subsection (i) for the nucleon). Then, the nonvanishing vector and tensor polarizations which are accessible from elastic electron scattering from a polarized target are given in the Madison convention by [36]

$$p_z = \sqrt{2/3} t_{10} = (\sqrt{2}/3) \frac{R_1^0}{f_1}, \quad (3.16a)$$

$$p_x = -(2/\sqrt{3}) t_{11} = (\sqrt{2}/3) \frac{R_1^1}{f_1}, \quad (3.16b)$$

$$p_{zz} = \sqrt{2} t_{20} = \sqrt{2/3} \frac{R_2^0}{f_2}, \quad (3.16c)$$

$$p_{xz} = -\sqrt{3} t_{21} = \sqrt{3/2} \frac{R_2^1}{f_2}, \quad (3.16d)$$

and

$$p_{xx} - p_{yy} = 2\sqrt{3}t_{22} = 2\sqrt{6}\frac{R_2^2}{f_2}. \quad (3.16e)$$

Note that the expression for p_{zz} in terms of t_{20} (T_{20} in their notation) as given in [14] is off by a sign; also, [63] should have a factor of $\sqrt{3}N$ rather than $\sqrt{2}N$ in their Eq. (4b).

The three form factors depend directly on the detailed behaviour of the deuteron wavefunction, and thus a measurement of their values over an extended range of momentum transfer may place some restrictions on possible models for the nucleon–nucleon interaction. Also, the effects of relativistic corrections and meson-exchange currents can be probed at high momentum transfer, where they are expected to be important; in addition, the transition from the meson–baryon description of the nucleus to the quark–gluon description may be observable at such momentum transfers. An experiment measuring the recoil t_{20} tensor polarization of the deuteron has recently been performed at the Bates Laboratory with an unpolarized low duty-factor electron beam using a water target and a conventional ${}^3\text{He}(\vec{d}, p)$ polarimeter; the first results from this experiment are shown in Fig. 6 [64]. It is hoped that the development of improved techniques, especially higher energy polarimeters [22] and higher duty-factor beams, will make it possible to extend such measurements out to the interesting higher momentum transfer region where the different state-of-the-art NN potentials lead to values of t_{20} which differ significantly from one another.

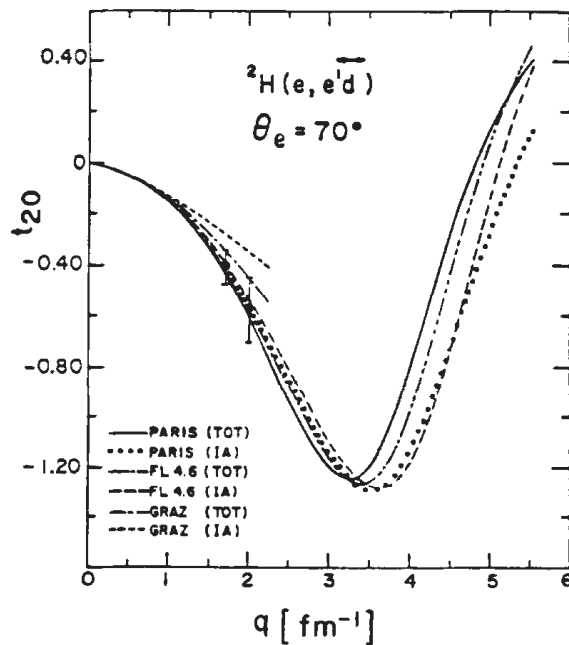


FIG. 6. Recoil t_{20} tensor polarization for elastic electron scattering from the deuteron (from [64]).