Elastic Scattering Fit Function

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In this short report I will briefly explain how I fit the elastic data and calculate the beam energy. The momentum (energy) of the scattered electrons detected in the spectrometer is given by:

$$E' = p_c(1+\delta) + \Delta E_1 = \frac{E_0 - \Delta E_0}{1 + \frac{E_0 + \Delta E_0}{M}(1-\cos\theta)}, \tag{1}$$

where p_c is the spectrometer constant (i.e. central momentum of the spectrometer), E_0 is the true beam energy and θ is the scattering angle. We also need to consider that incoming electron loses some energy in the target before the reaction and that outgoing scattered electron loses its energy on its way out of the target. I have labeled these two energy losses with ΔE_0 and ΔE_1.

We will use formula (1) to fit our experimental data. With it we would like to determine the beam energy for the given set of experimental data. However, before doing that we need to consider that beam energy was not constant but was changing during the whole run period. In order to calculate the true value of the beam energy, we need to correct our measurements for these small variations in beam energy. We have done that by introducing additional term ΔE_T, which represents the beam energy difference between a current and a golden run. This in the end gives us:

$$E' = p_c(1+\delta) + \Delta E_1 = \frac{E_0 + \Delta E_T - \Delta E_0}{1 + \frac{E_0 + \Delta E_0}{M}(1-\cos\theta)}, \tag{2}$$

We have estimated the beam energy difference ΔE_T from the Tiefenbach data. We have chosen a golden run (the choice is of course arbitrary) and set its Tiefenbach value as a golden one. Then we calculated all energy differences ΔE_T relative to this value.

Now we can go a step further and try to modify equation (2) a bit in order to simplify our fitting procedure. If we would use it in current form we would have to fit our data point-by-point, which is a bit more difficult than fitting a continuous function. Since energy losses are $\Delta E_{0,1} < 2\,\mathrm{MeV}$ and beam energy differences are also $\Delta E_T < 1\,\mathrm{MeV}$, which is much smaller than the beam energy, we can expand our equation (2) into Taylor series:

$$E' = \frac{E_0}{1 + \frac{E_0}{M}(1-\cos\theta)} + \frac{E_0\kappa}{1 + \frac{E_0}{M}(1-\cos\theta)} + \frac{\widehat{\Delta E_0}\kappa}{1 + \frac{E_0}{M}(1-\cos\theta)} + \frac{\widehat{\Delta E_0}\kappa}{1 + \frac{\widehat{\Delta E_0}\kappa}{M}(1-\cos\theta)}, \tag{3}$$

where $\widehat{\Delta E_0} = \Delta E_T + \Delta E_0$ and κ is defined as:

$$\kappa = \frac{\frac{\widehat{\Delta E}}{M}(1-\cos\theta)}{1 + \frac{E_0}{M}(1-\cos\theta)} \approx 3\times 10^{-5} \tag{4}$$

Equation (3) can be further written as:

$$
\begin{aligned}
E' &= \frac{E_0}{1+\frac{E_0}{M}(1-\cos\theta)} + \frac{E_0}{1+\frac{E_0}{M}(1-\cos\theta)}\frac{\widehat{\Delta E_0}}{M}(1-\cos\theta) + \widehat{\Delta E_0} - \frac{\widehat{\Delta E_0}E_0}{M}(1-\cos\theta) \\
&\approx \frac{E_0}{1+\frac{E_0}{M}(1-\cos\theta)} + E_0\frac{\widehat{\Delta E_0}}{M}(1-\cos\theta) - \underbrace{E0\frac{E_0}{M}(1-\cos\theta)\frac{\widehat{\Delta E_0}}{M}(1-\cos\theta)}_{\approx 2\times10^{-3}\,\mathrm{MeV}} \\
&\quad + \widehat{\Delta E_0} - \frac{\widehat{\Delta E_0}E_0}{M}(1-\cos\theta) \\
&\approx \frac{E_0}{1+\frac{E_0}{M}(1-\cos\theta)} + \widehat{\Delta E_0}
\end{aligned} \tag{5}
$$

In the end we can arrange our formula in following way:

$$
p_c(1+\delta) + \Delta E_1 + \Delta E_0 - \Delta E_T = \frac{E_0}{1+\frac{E_0}{M}(1-\cos\theta)} \tag{6}
$$

Equation (6) allows us to correct our measured data points for energy losses and changes in beam energy before fitting the data. This way we can use Root/Analyzer to fit points with the TGraph class and do not need to write our own fitting algorithm. If we would use equation (2) we would have to write our own program for the χ^2 minimization and determination of an error. This way Root does all the hard work for us.