

```
In[1]:= Needs["ErrorBarPlots`"]
In[2]:= SetDirectory[NotebookDirectory[]]
Out[2]:= /Users/miham/Desktop/ISR/RadiusFit
```

Data

```
In[3]:= M = 0.93827;
mup = 2.79;
ħc = 0.197327;
```

```
In[6]:= Tau[Q2_] :=  $\frac{Q2}{4 M^2}$ 
```

```
In[7]:= Eps[th_, Q2_] :=  $\left(1 + 2 (1 + \text{Tau}[Q2]) \text{Tan}\left[\frac{\text{th}}{2}\right]^2\right)^{-1}$ 
```

```
In[8]:= GD[x_, a_, n_] :=  $\frac{1}{\left(1 + \frac{x}{a}\right)^2} * n$ 
```

```
In[9]:= GM[Q2_, R_, n_, a_, b_] :=  $n \left(1 - \frac{1}{6 * \hbar c^2} R^2 Q2 + \frac{a}{120 * \hbar c^4} Q2^2 - \frac{b}{5040 * \hbar c^6} Q2^3\right)$ 
```

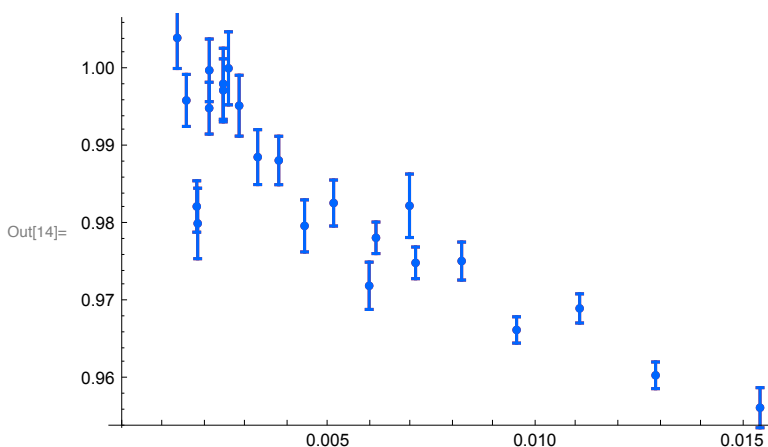
```
In[10]:= data2 = Select[Map[{{#[[1]], #[[2]], #[[3]]} &,
Import["ISRFormFactorNoEl.dat"]], #[[1]] < 0.03 &];
```

```
In[11]:=
```

```
In[12]:= data2;
```

```
In[13]:= data = Join[Select[data2, (#[[1]] > 0.0 && #[[1]] < 0.1) &];];
```

```
In[14]:= s1 = Show[ErrorListPlot[Map[{{#[[1]], #[[2]]}, ErrorBar[#[[3]]]} &, data],
PlotStyle -> Hue[0]], ErrorListPlot[
Map[{{#[[1]], #[[2]]}, ErrorBar[#[[3]]]} &, data2], PlotStyle -> Hue[0.6]]]
```



Manual Fit with Chi2

```
In[15]:= Chi2b[R_, n_, a_, b_, data_] :=
  
$$\frac{1}{\text{Length}[data] - 2} \text{Sum} \left[ \left( \text{GM}[data[[i, 1]], R, 1, a, b] - n * data[[i, 2]] \right)^2 / \right. \\ \left. (n^2 * data[[i, 3]]^2), \{i, 1, \text{Length}[data]\} \right]$$

```

```
In[16]:= min = FindMinimum[Chi2b[RR, nnn, 2.59, 29.8, data], {RR, nnn}]
```

FindMinimum::stol:

The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

```
Out[16]:= {2.99202, {RR -> 0.855357, nnn -> 1.00143}}
```

```
In[17]:= Clear[r]
```

```
In[18]:= alpha2[data_, RR_, nnn_, a_, b_] :=
  Sum[ 
$$\frac{1}{data[[i, 3]]^2} \left\{ \left\{ D[GM[Q2, r, \frac{1}{c}, a, b], r] D[GM[Q2, r, \frac{1}{c}, a, b], r], \right. \right. \\ D[GM[Q2, r, \frac{1}{c}, a, b], r] D[GM[Q2, r, \frac{1}{c}, a, b], c] \left. \right\}, \\ \left\{ D[GM[Q2, r, \frac{1}{c}, a, b], c] D[GM[Q2, r, \frac{1}{c}, a, b], r], \right. \\ \left. D[GM[Q2, r, \frac{1}{c}, a, b], c] D[GM[Q2, r, \frac{1}{c}, a, b], c] \right\} \left. \right\} /. \\ \{r -> RR, c -> nnn, Q2 -> data[[i, 1]]\}, \{i, 1, \text{Length}[data]\}]$$

```

```
In[19]:= RadiusGM2[{a_, n_}, cc_] := Module[{},
  dd = D[GM[x, R, 1, 2.59, 29.8], x] /. {x -> 0};
  ra = Sqrt[-6 * 0.197326^2 * dd];
  dr = Sqrt[(D[ra, R])^2 cc[[1, 1]]];
  {ra, dr, n1, Sqrt[cc[[2, 2]]]} /. {R -> a, n1 -> n}
]
```

```
In[20]:= FindMyRadius[a_, b_, data_] := Module[{},
  min = FindMinimum[Chi2b[RR, nnn, a, b, data], {RR, nnn}];
  (*Print[min];*)
  Cov = Inverse[alpha2[data, RR, nnn, a, b] /. min[[2]]];
  (*Print[Cov];*)
  RadiusGM2[{RR, nnn} /. min[[2]], Cov]
]
```

```
In[126]:= res = FindMyRadius[2.59, 29.8, data]
```

FindMinimum::stol:

The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

```
Out[126]:= {0.855353, 0.0191881, 1.00143, 0.00119756}
```

Radius Model dependent Error band

```
In[22]:= ModelDepRadiusError[a_, da_, b_, db_, NN_] := Module[{},
  alist = RandomVariate[NormalDistribution[a, da], NN];
  blist = RandomVariate[NormalDistribution[b, db], NN];
  fulllist = Table[{alist[[i]], blist[[i]]}, {i, 1, NN}];
  ll = Map[FindMyRadius[#[[1]], #[[2]], data] &, fulllist];
  meanr = Mean[Map[#[[1]] &, ll]];
  sigmar = Sqrt[Variance[Map[#[[1]] &, ll]]];
  meann = Mean[Map[#[[3]] &, ll]];
  sigman = Sqrt[Variance[Map[#[[3]] &, ll]]];
  {meanr, sigmar, meann, sigman}
]
```

```
In[23]:= ModelDepRadiusError[2.59, 0.194, 29.8, 14.71, 100]
```

FindMinimum::Istol:

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The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

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The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

General::stop: Further output of FindMinimum::Istol will be suppressed during this calculation >>

```
Out[23]= {0.854989, 0.00248736, 1.00144, 0.0000585938}
```

```
In[24]:=
```

Manual Form - Factor Error band

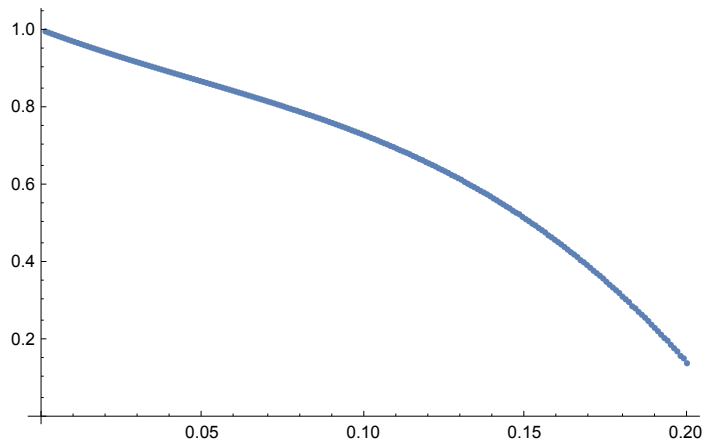
```
In[25]:= FormFactor[Q2_, NN_, R_, dR_, a_, da_, b_, db_] := Module[{},
  rList = RandomVariate[NormalDistribution[R, dR], NN];
  aList = RandomVariate[NormalDistribution[a, da], NN];
  bList = RandomVariate[NormalDistribution[b, db], NN];
  fulllist = Table[{rList[[i]], aList[[i]], bList[[i]]}, {i, 1, NN}];
  FF = Map[GM[Q2, #[[1]], 1, #[[2]], #[[3]]] &, fulllist];
  mean = Mean[FF];
  sigma = Sqrt[Variance[FF]];
  {Q2, mean, sigma}
]
```

```
In[28]:= FormFactor[0.01, 100, res[[1]], res[[2]], 2.59, 0.194, 29.8, 14.71]
```

```
Out[28]= {0.01, 0.969967, 0.00139184}
```

```
FFTable = Table[FormFactor[Q2, 100 000, res[[1]],
  res[[2]], 2.59, 0.194, 29.8, 14.71], {Q2, 0.001, 0.2, 0.001}];
```

```
ListPlot[Map[#[[1]], #[[2]]] &, FFTable]
```



```
Export["ResultsISRStandAloneManualNew.dat", FFTable];
```

Systematic Uncertainty

495 MeV

```
In[143]:= Syst495MeVData = Import["ISRFFSystematicalError495MeV.dat"]
s495 = Interpolation[Map[#[[1]], #[[2]] / 100] &, Syst495MeVData];

Out[143]= {{0.0170417, 0.445293, 0}, {0.0153986, 0.444452, 0},
           {0.0128783, 0.418822, 0}, {0.0110455, 0.393569, 0}, {0.0095245, 0.37142, 0},
           {0.00819877, 0.389644, 0}, {0.00708697, 0.559909, 0}, {0.0061315, 0.670987, 0}}

In[145]:= s495[0.01]

Out[145]= 0.00375348

In[146]:= Data495MeV = Select[Map[#[[1]], #[[2]], #[[3]]] &,
                               Import["ISRFormFactor495MeV.dat"], #[[1]] < 0.017 &]

Out[146]= {{0.0153986, 0.956278, 0.0025899},
           {0.0128783, 0.960465, 0.00172386}, {0.0110455, 0.96911, 0.00188524},
           {0.0095245, 0.966328, 0.0016995}, {0.00819877, 0.97523, 0.00245552},
           {0.00708697, 0.975, 0.00204589}, {0.0061315, 0.978237, 0.00203728}}

In[147]:= Combined495MeV = Map[#[[1]], #[[2]], #[[3]], s495[#[[1]]] &, Data495MeV]

Out[147]= {{0.0153986, 0.956278, 0.0025899, 0.00444452},
           {0.0128783, 0.960465, 0.00172386, 0.00418822},
           {0.0110455, 0.96911, 0.00188524, 0.00393569},
           {0.0095245, 0.966328, 0.0016995, 0.0037142},
           {0.00819877, 0.97523, 0.00245552, 0.00389644},
           {0.00708697, 0.975, 0.00204589, 0.00559909},
           {0.0061315, 0.978237, 0.00203728, 0.00670987}}
```

330 MeV

```
In[51]:= Syst330MeVData = Import["ISRFFSystematicalError330MeV.dat"]
s330 = Interpolation[Map[#[[1]], #[[2]] / 100] &, Syst330MeVData];
Out[51]= {{0.0078145, 1.09457, 0}, {0.0069408, 1.08763, 0}, {0.00596758, 1.02248, 0},
{0.00510955, 0.96166, 0}, {0.00440878, 0.900458, 0}, {0.0037858, 0.877308, 0},
{0.00327766, 0.85083, 0}, {0.00283015, 0.790093, 0}, {0.00244682, 0.803002, 0},
{0.00244627, 0.802977, 0}, {0.00210893, 0.868517, 0}, {0.00182937, 0.878239, 0}}

In[54]:= s330[0.005]
Out[54]= 0.00951384

In[56]:= Data330MeV = Select[Map[#[[1]], #[[2]], #[[3]]] &,
Import["ISRFormFactor330MeV.dat"], #[[1]] < 0.007 &]
Out[56]= {{0.0069408, 0.982376, 0.00410094},
{0.00596758, 0.972034, 0.00305643}, {0.00510955, 0.982733, 0.00297362},
{0.00440878, 0.979775, 0.00336679}, {0.0037858, 0.988231, 0.00312875},
{0.00327766, 0.988665, 0.00354716}, {0.00283015, 0.99531, 0.00393255},
{0.00244682, 0.997295, 0.0040697}, {0.00244627, 0.99814, 0.00459216},
{0.00210893, 0.999881, 0.00404735}, {0.00182937, 0.98009, 0.00455452}}

In[57]:= Combined330MeV = Map[#[[1]], #[[2]], #[[3]], s330[#[[1]]]] &, Data330MeV]
Out[57]= {{0.0069408, 0.982376, 0.00410094, 0.0108763},
{0.00596758, 0.972034, 0.00305643, 0.0102248},
{0.00510955, 0.982733, 0.00297362, 0.0096166},
{0.00440878, 0.979775, 0.00336679, 0.00900458},
{0.0037858, 0.988231, 0.00312875, 0.00877308},
{0.00327766, 0.988665, 0.00354716, 0.0085083},
{0.00283015, 0.99531, 0.00393255, 0.00790093},
{0.00244682, 0.997295, 0.0040697, 0.00803002},
{0.00244627, 0.99814, 0.00459216, 0.00802977},
{0.00210893, 0.999881, 0.00404735, 0.00868517},
{0.00182937, 0.98009, 0.00455452, 0.00878239}}
```

195 MeV

```
In[58]:= Syst195MeVData = Import["ISRFFSystematicalError195MeV.dat"]
s195 = Interpolation[Map[#[[1]], #[[2]] / 100] &, Syst195MeVData];
Out[58]= {{0.00273397, 1.02731, 0}, {0.00257314, 1.00992, 0}, {0.00210662, 0.991071, 0},
{0.00180739, 0.966454, 0}, {0.001556, 1.02105, 0}, {0.00133657, 1.01279, 0}}

In[61]:= s195[0.002]
Out[61]= 0.00976497

In[65]:= Data195MeV = Select[Map[#[[1]], #[[2]], #[[3]]] &,
Import["ISRFormFactor195MeV.dat"], #[[1]] < 0.0027 &]
Out[65]= {{0.00257314, 1.00014, 0.00471763},
{0.00210662, 0.994993, 0.00334811}, {0.00180739, 0.982284, 0.00332201},
{0.001556, 0.99599, 0.00336262}, {0.00133657, 1.00407, 0.0039531}}
```

```
In[66]:= Combined195MeV = Map[#{#[1]}, #[2]], #[3]], s195[#[1]]] &, Data195MeV]
```

```
Out[66]:= {{0.00257314, 1.00014, 0.00471763, 0.0100992},
           {0.00210662, 0.994993, 0.00334811, 0.00991071},
           {0.00180739, 0.982284, 0.00332201, 0.00966454},
           {0.001556, 0.99599, 0.00336262, 0.0102105},
           {0.00133657, 1.00407, 0.0039531, 0.0101279}}
```

```
In[148]:= CombinedAll = Join[Combined495MeV, Combined330MeV, Combined195MeV];
```

Analysis

```
In[153]:= RandomizeData[data_] := Module[{},
      NewData =
        Map[#{#[1]}, RandomVariate[NormalDistribution[#[[2]], #[[4]], 1][[1]],
           #[[3]]] &, data];
      NewData
    ]
```

```
In[154]:= res = FindMyRadius[2.59, 29.8, data]
```

FindMinimum::lstol:

The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

```
Out[154]:= {0.855353, 0.0191881, 1.00143, 0.00119756}
```

```
In[136]:= Length[data]
```

```
Out[136]:= 23
```

```
In[151]:= Length[RandomizeData[CombinedAll]]
```

```
Out[151]:= 23
```

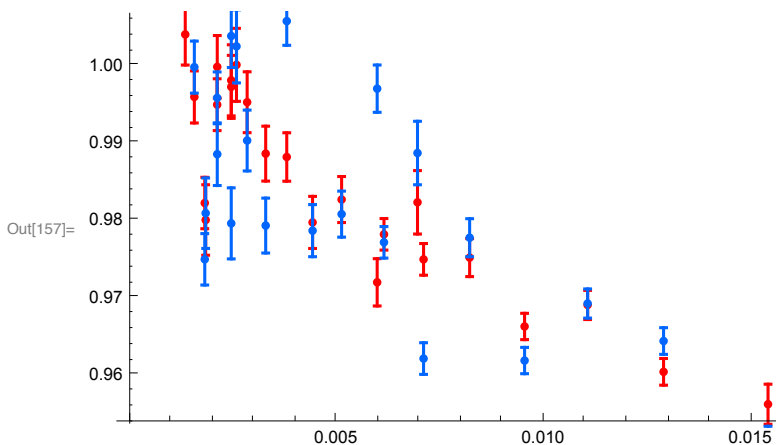
```
In[156]:= FindMyRadius[2.59, 29.8, RandomizeData[CombinedAll]]
```

FindMinimum::lstol:

The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

```
Out[156]:= {0.812525, 0.0202958, 1.00394, 0.00120357}
```

```
In[157]:= Show[ErrorListPlot[Map[#{#[1]}, #[2]], ErrorBar[#[[3]]] &, CombinedAll],
      PlotStyle -> Hue[0]], ErrorListPlot[Map[#{#[1]}, #[2]], ErrorBar[#[[3]]] &,
      RandomizeData[CombinedAll]], PlotStyle -> Hue[0.6]]
```



```
In[158]:= SysPoints = Table[
  Join[{i}, FindMyRadius[2.59, 29.8, RandomizeData[CombinedAll]]], {i, 1, 500}];
```

FindMinimum:lstol:

The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

FindMinimum:lstol:

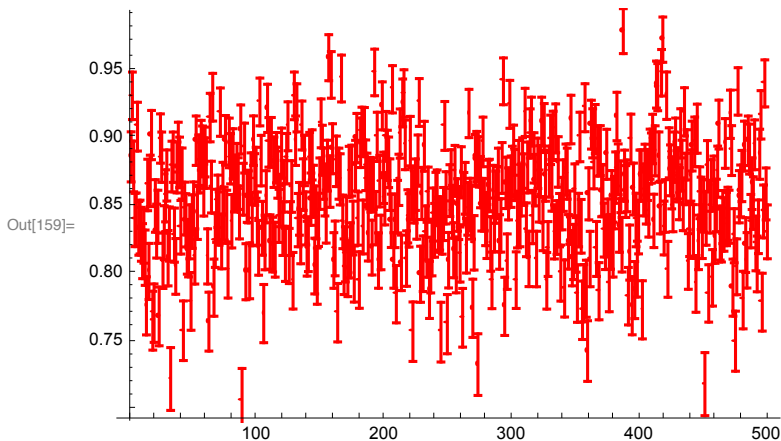
The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

FindMinimum:lstol:

The line search decreased the step size to within the tolerances specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances >>

General::stop: Further output of FindMinimum:lstol will be suppressed during this calculation >>

```
In[159]:= ErrorListPlot[
  Map[{{#[[1]], #[[2]]}, ErrorBar[#[[3]]]} &, SysPoints], PlotStyle -> Hue[0]]
```

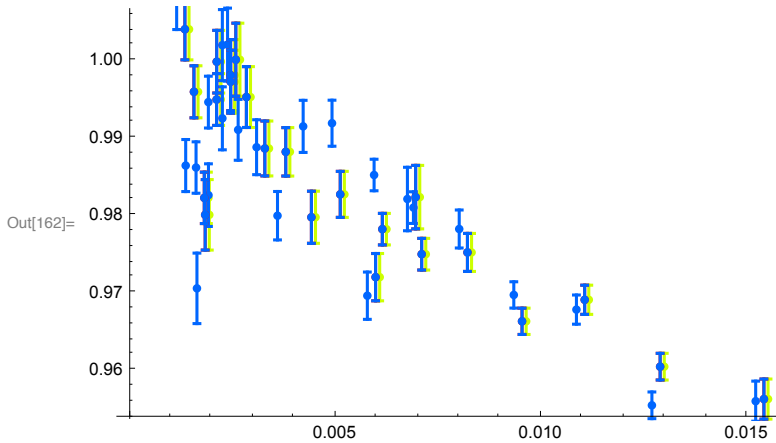


```
In[160]:= mean = Mean[Map[#[[2]] &, SysPoints]]
sigma = Sqrt[Variance[Map[#[[2]] &, SysPoints]]]
```

Out[160]= 0.852881

Out[161]= 0.042325

```
In[162]:= Show[
  ErrorListPlot[Map[{{#[[1]] + 0.0001, #[[2]]}, ErrorBar[#[[3]]]} &, CombinedAll],
    PlotStyle -> Hue[0.2]],
  ErrorListPlot[Map[{{#[[1]] - 0.0002, #[[2]]}, ErrorBar[#[[3]]]} &,
    RandomizeData[CombinedAll]], PlotStyle -> Hue[0.6]],
  ErrorListPlot[Map[{{#[[1]], #[[2]]}, ErrorBar[#[[3]]]} &, data],
    PlotStyle -> Hue[0]], ErrorListPlot[
  Map[{{#[[1]], #[[2]]}, ErrorBar[#[[3]]]} &, data2], PlotStyle -> Hue[0.6]]]
```



```
In[163]:= Integrate[ $\frac{t}{x} \left(\frac{x}{F}\right)^t$ , {x, 0, y}]
```

```
Out[163]:= ConditionalExpression[ $\left(\frac{y}{F}\right)^t$ , Re[t] > 0]
```

```
In[169]:=  $\left(\frac{0.00002}{0.5}\right)^{0.02}$ 
```

```
Out[169]:= 0.81666
```

```
In[170]:=  $\left(\frac{0.00002}{0.3}\right)^{0.02}$ 
```

```
Out[170]:= 0.825046
```

```
In[176]:=  $\left(\frac{0.000006}{0.2}\right)^{0.002}$ 
```

```
Out[176]:= 0.979387
```